Percolação de longo alcance e renormalização

Augusto Teixeira

Instituto Nacional de Matemática Pura e Aplicada
Rio de Janeiro - RJ

Avanços e Perspectivas, Novembro 2012

Colaboração com
Vladas Sidoravicius (IMPA) e Marcelo Hilário (UFMG)

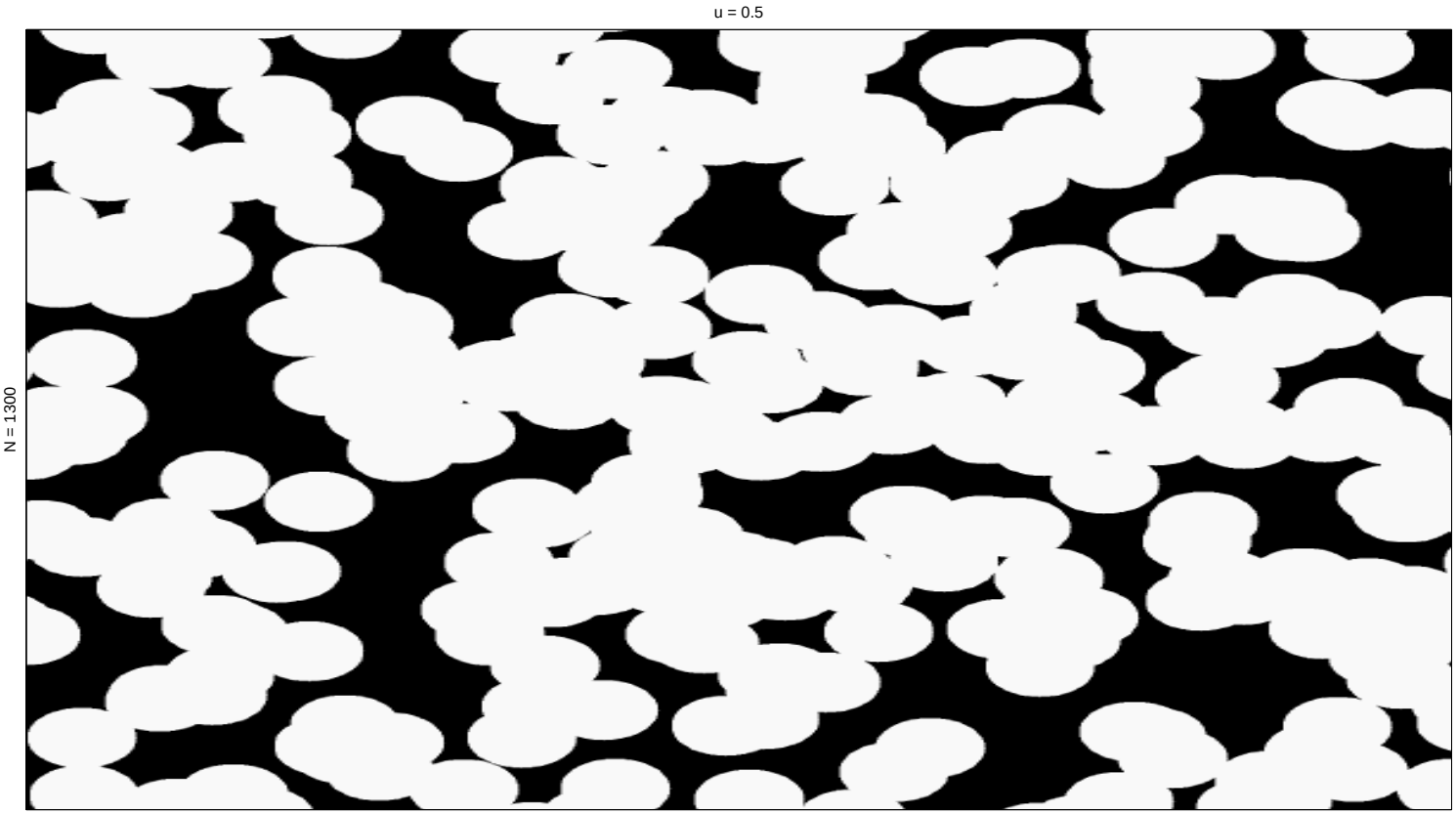
Percolação

- Corrosão, percolação em meios porosos, redes sem fio...
- Modelos bastante simples
- Transição de fase
- Universalidade

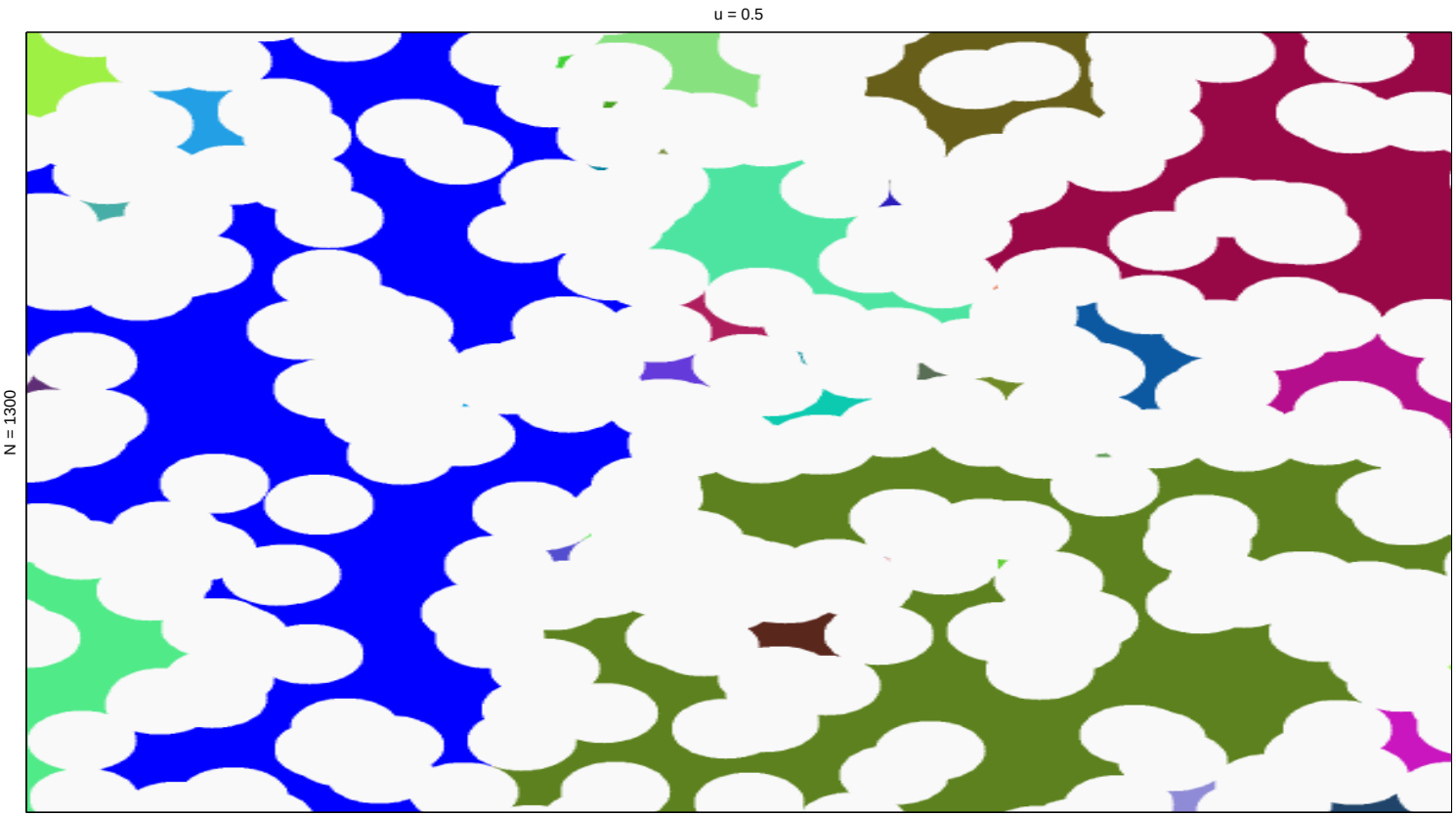
***** *Hammersley, Peierls, Harris, Kesten, Russo, Mandelbrot, Fisher, Aizenman, Chayes, Chayes, Frölich, Newman, Lebowitz, Barsky, van den Berg, Grimmett, Durrett, Menshikov, Slade, Hara, Benjamini, Cardy, Schramm, Werner, Smirnov...*

Percolação contínua

... ou Poissoniana, ou booleana.

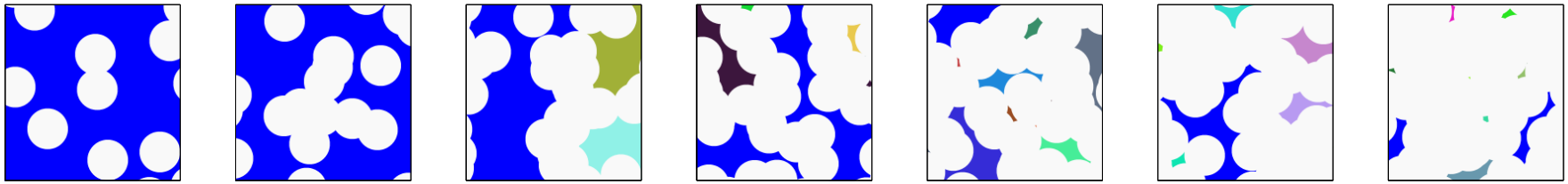


Conectividade após corrosão (uma cor pra cada componente)



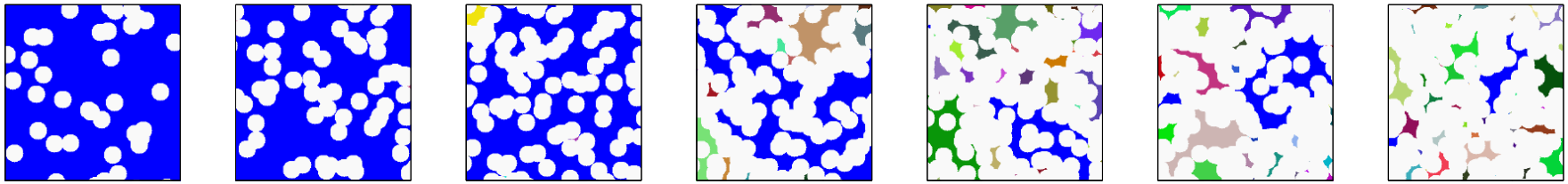
Modelo de Poisson

Densidade dos obstáculos



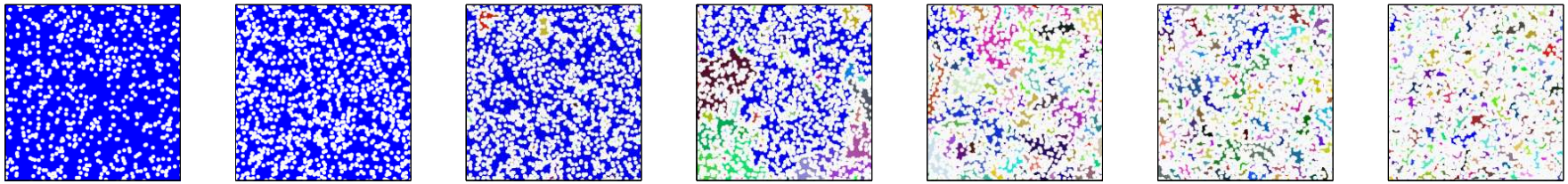
Modelo de Poisson

Aumentar o tamanho da amostra



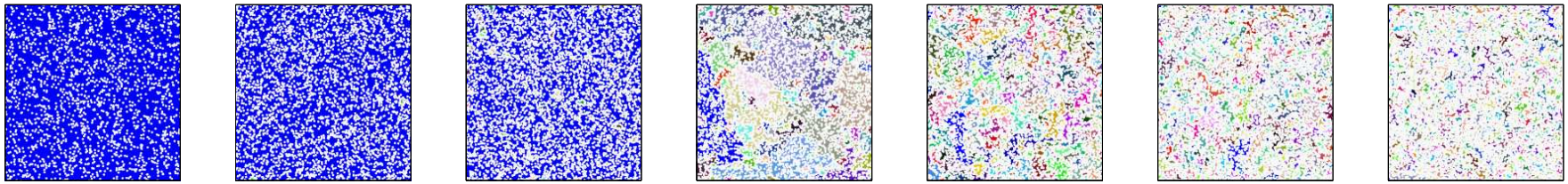
Modelo de Poisson

...mais...



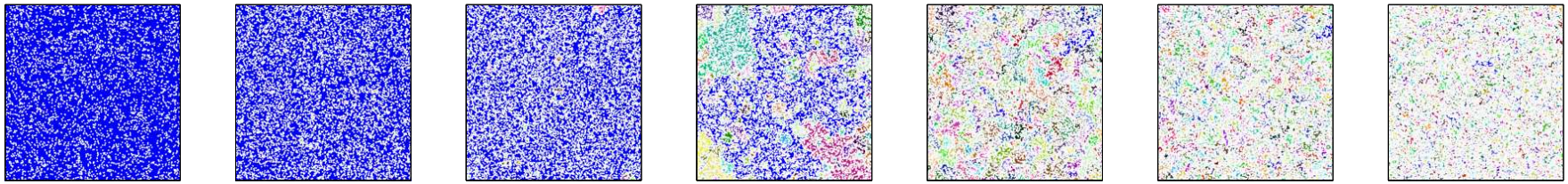
Modelo de Poisson

...mais...

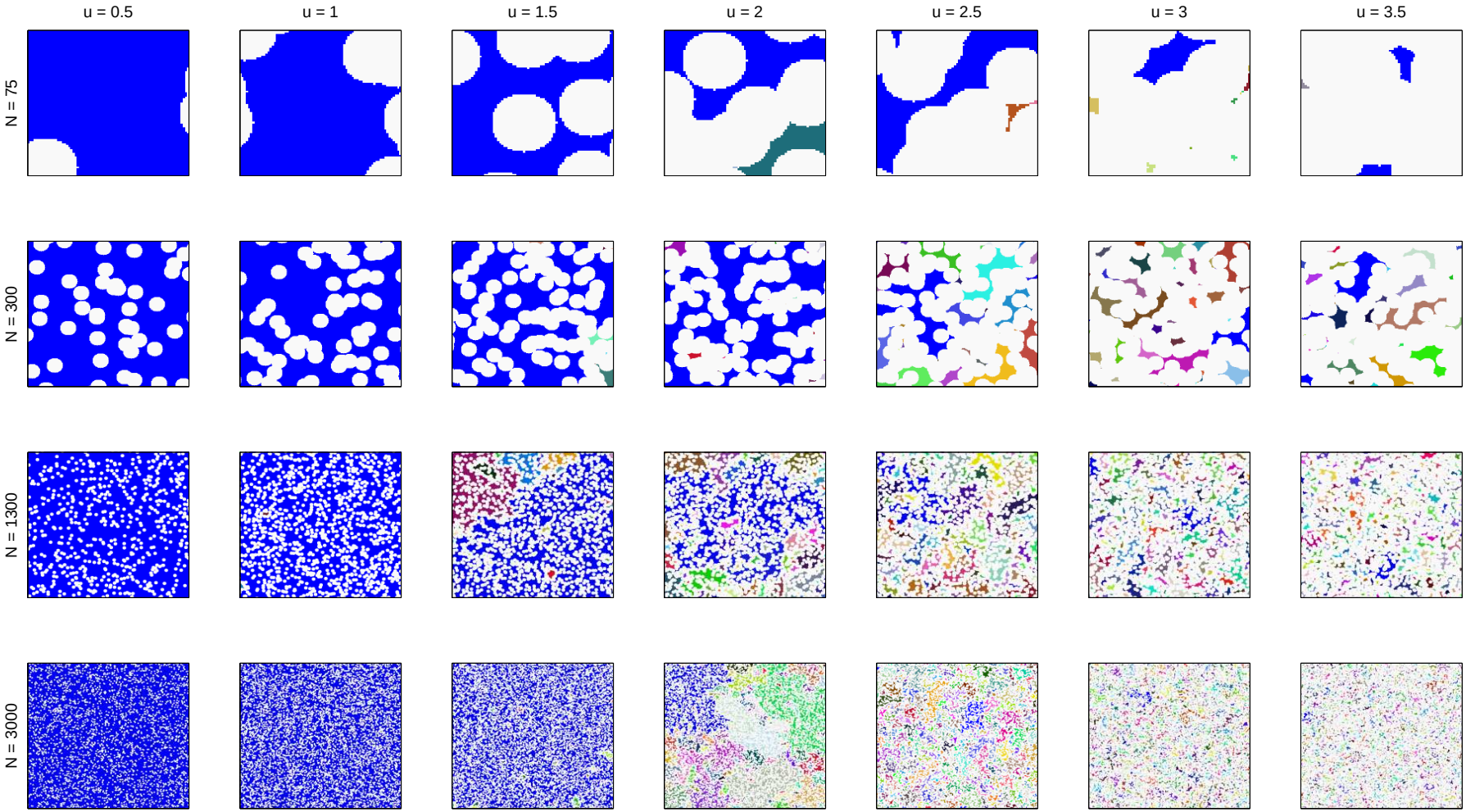


Modelo de Poisson

...“Transição de fase”



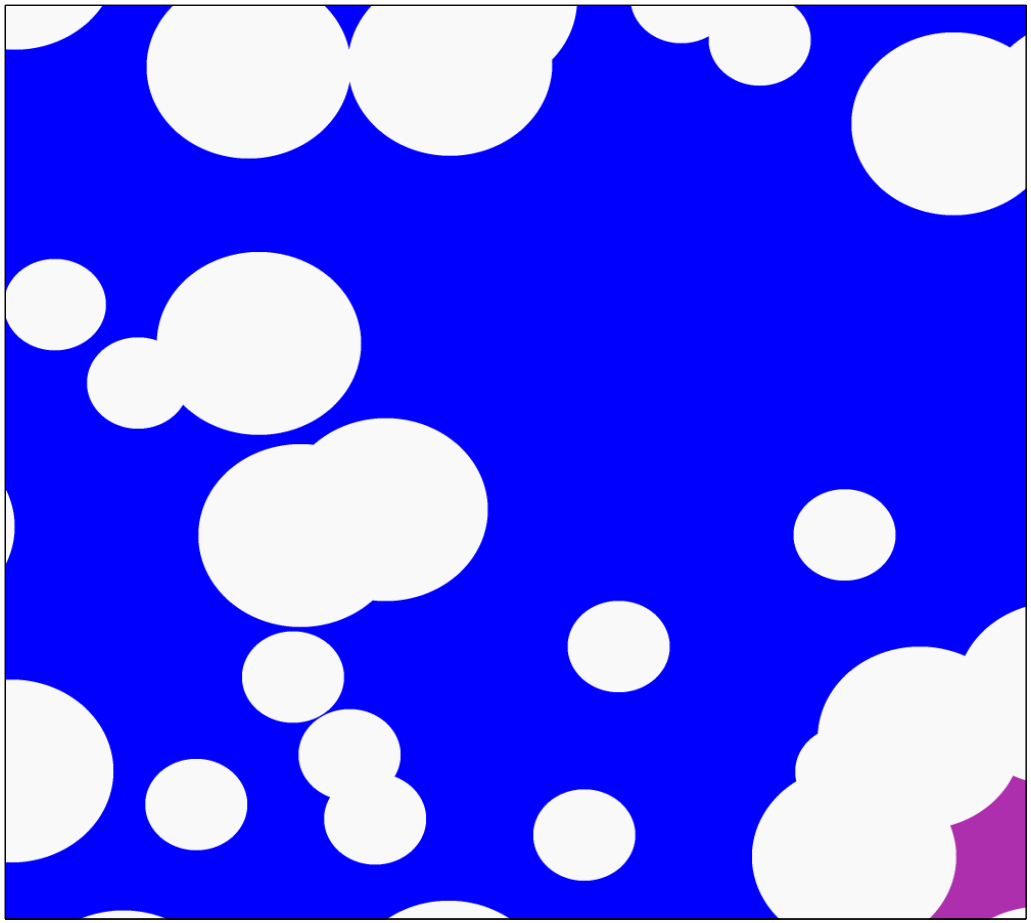
Modelo de Poisson



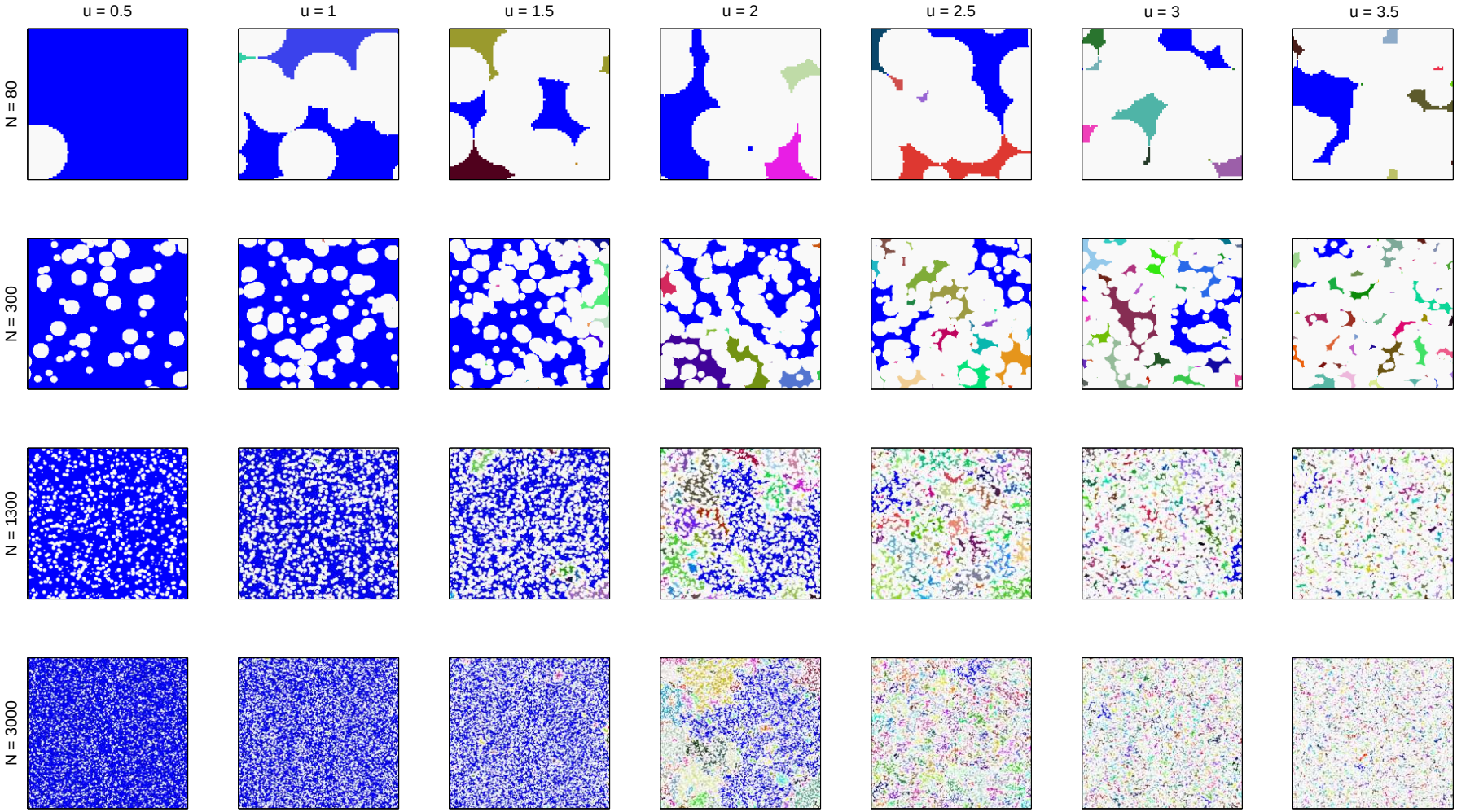
Transição de fase

- Mudança abrupta
- Sólido-líquido-gasoso
- Leis polinomiais
- Limites de escala

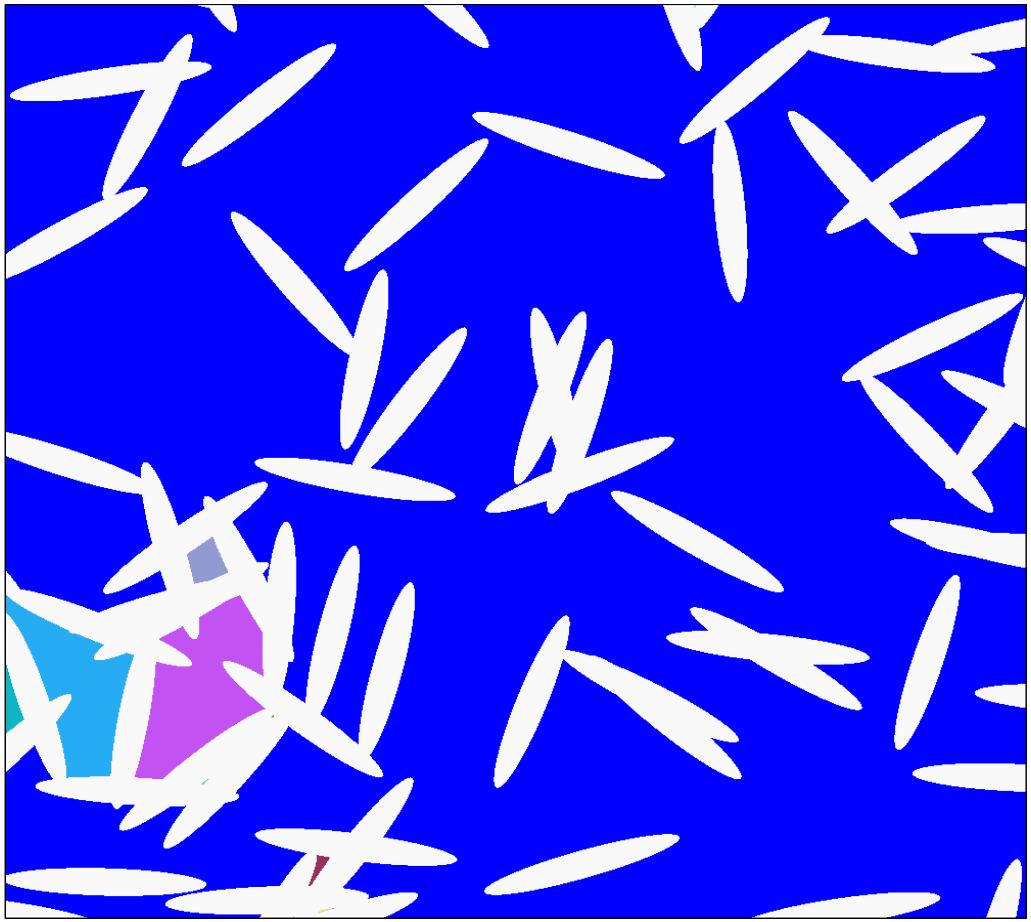
Raios aleatórios



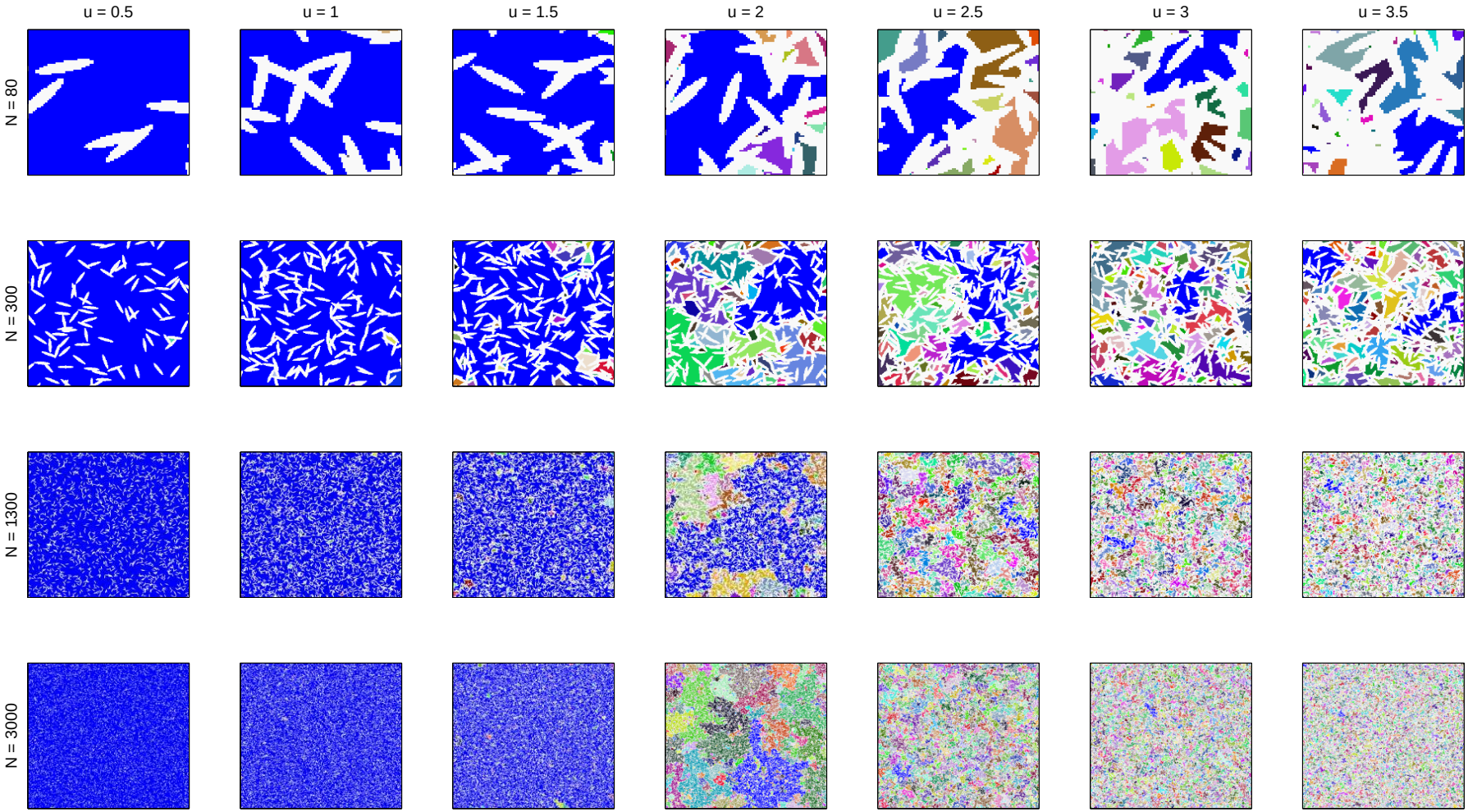
Raios aleatórios



Elipses

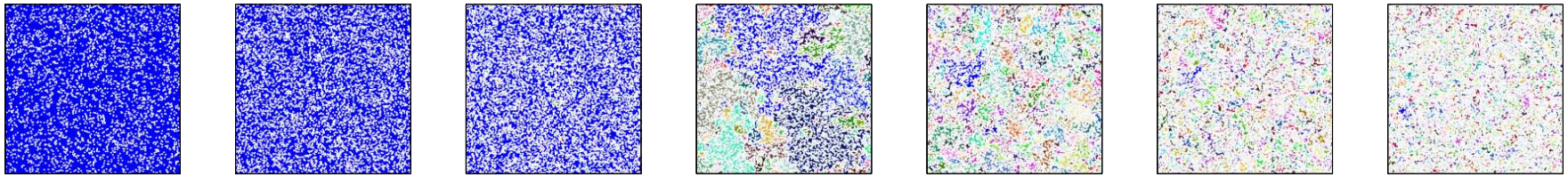


Elipses

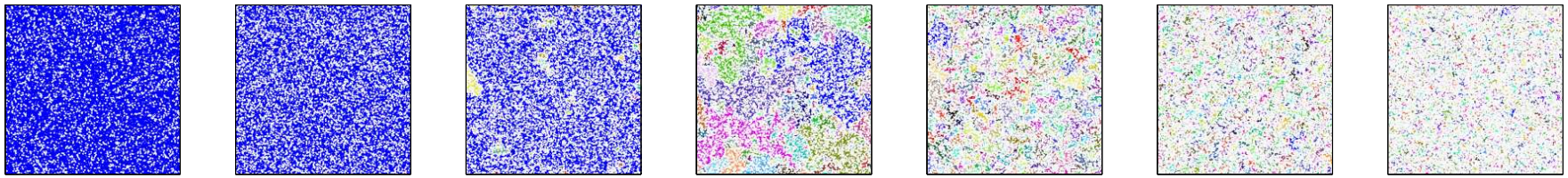


Universalidade

Bolas



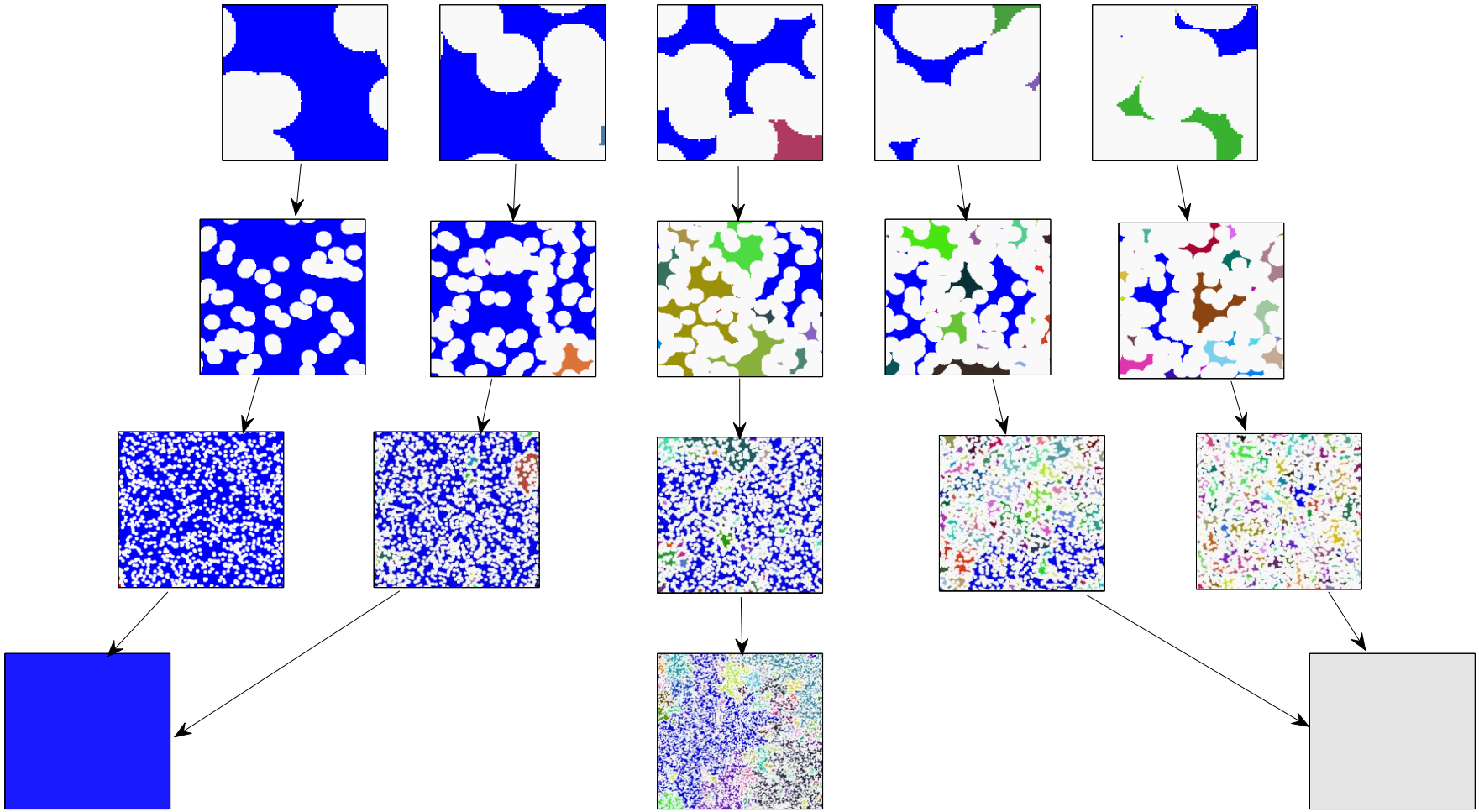
Raios aleatórios



Elipses



Grupo de renormalização



Ajuda a explicar

- Comportamento universal
- Comportamento crítico (pontos fixos)
- Expoentes críticos

Porém, muito difícil de estudar rigorosamente (matematicamente).

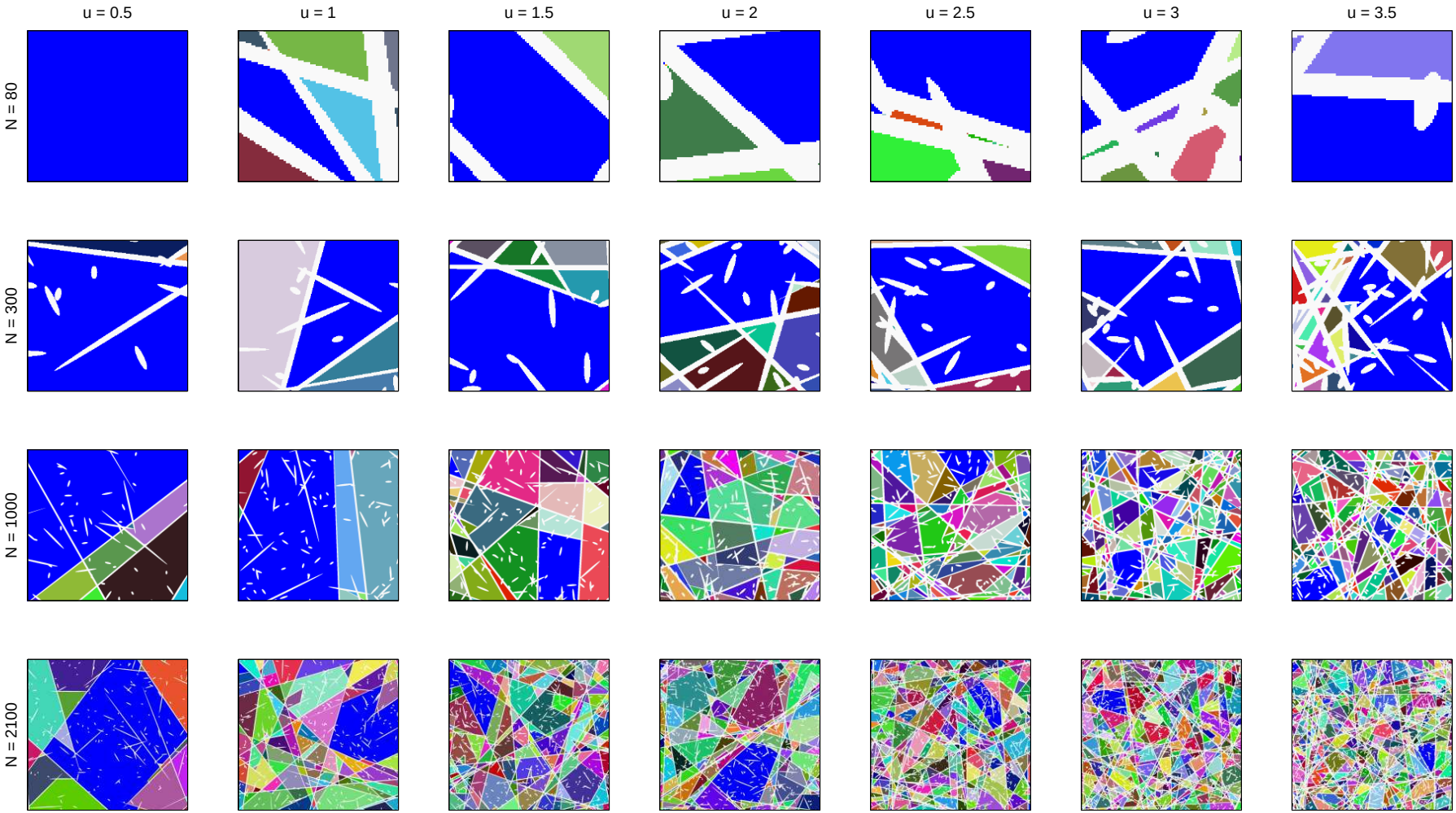
Exceção: Próximo aos pontos fixos degenerados (azul e branco)

Problema

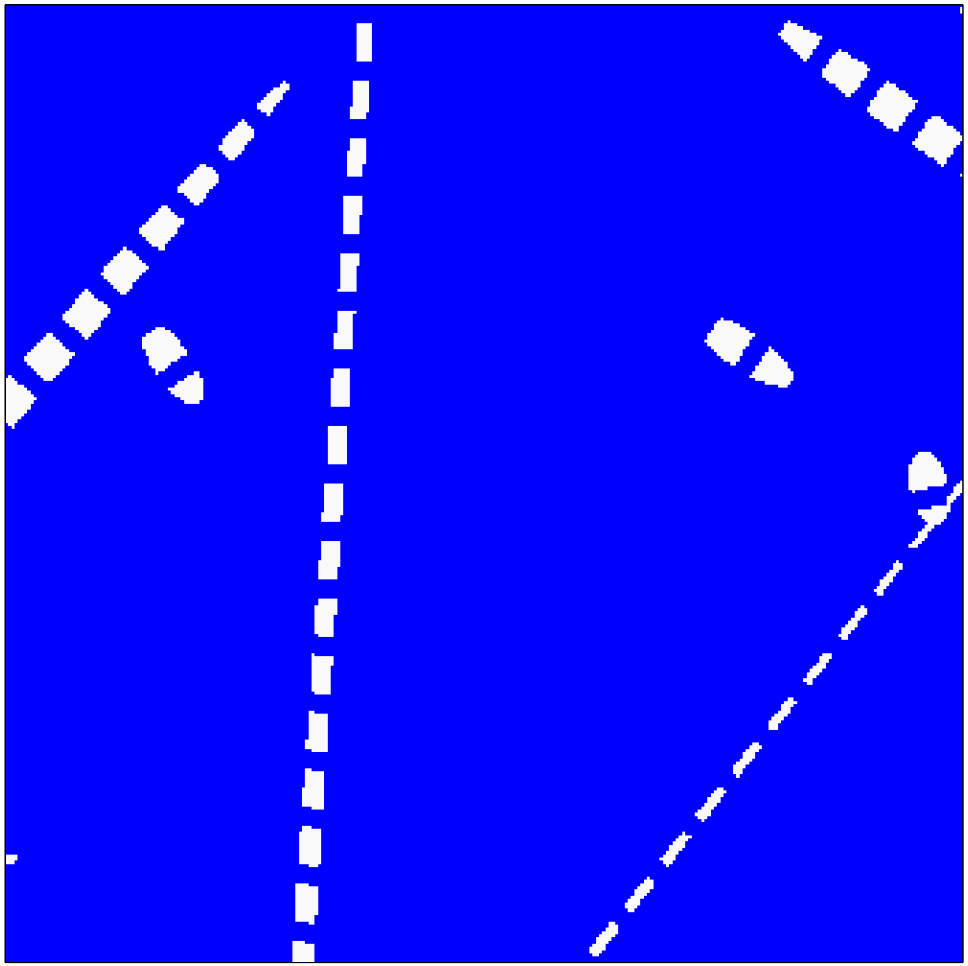
Percolação de longas elipses (J. Tykesson and D. Windisch)



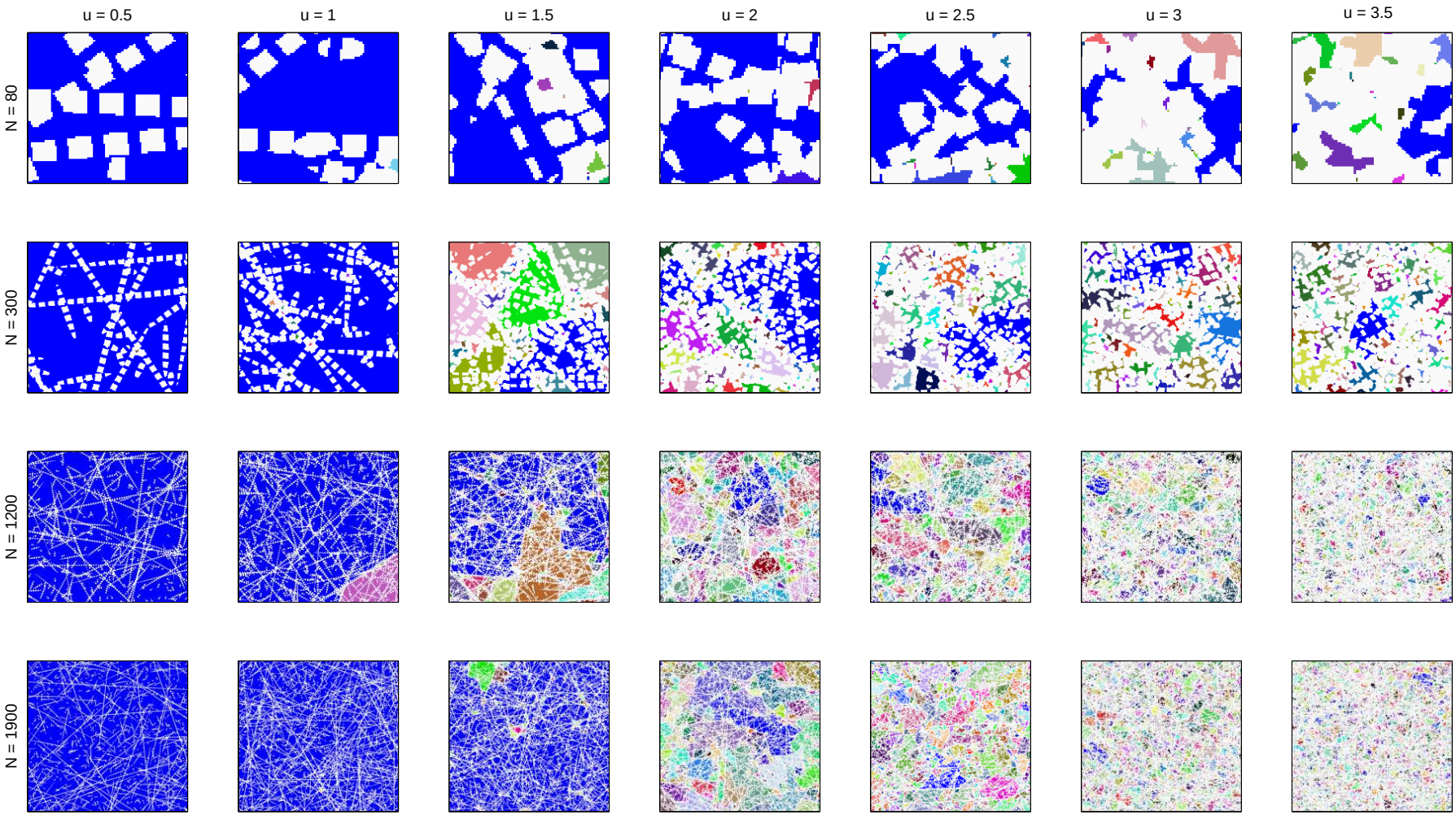
Longas elipses



Longas elipses picotadas



Comportamento “típico” recuperado



Conclusões e perspectivas

- Esforço de formalização recompensado
- Novas perspectivas: percolação de alcance muito longo
- Santo graal: tornar renormalização rigorosa perto do ponto crítico


A.S. Sznitman
Vacant set of random interlacements and percolation
to appear in the *Annals of Mathematics* (2007)


J. Tykesson and D. Windisch
Percolation in the vacant set of Poisson cylinders
to appear in the *Probability Theory and Related Fields* (2011)


M. Hilário, V. Sidoravicius, A. Teixeira
Cylinder's percolation in three dimensions
preprint (2012)

Obrigado!